Introduction

In this course I would like to introduce you to the basics of anyons and how to describe them in terms of topological quantum field theory (TQFT).
I will focus primarily on the more general case of non-abelian anyons, using Fibonacci anyons as a concrete example.

Plan

Lecture 1:
- Background & Motivation.
- Bosons & fermions revisited.
- (Simplified) Anyon statistics, fusion & braiding
- Fibonacci anyon example.

Lecture 2:
- TQFT
  - anyons
  - fusion
  - F-moves & Pentagon equation
  - R-moves & Hexagon equation.
  - Topological invariants.

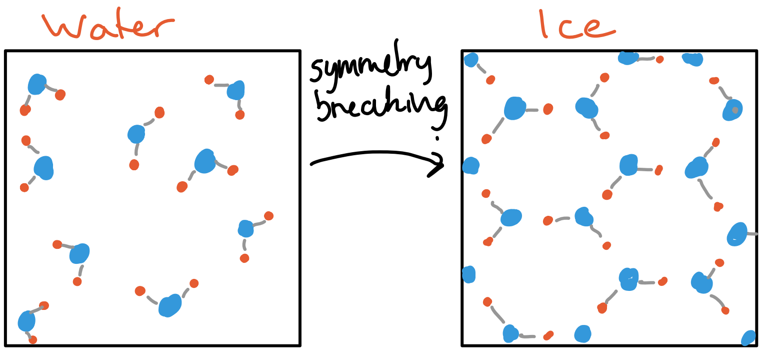
Anyons and topological order.

Anyons are particles that are neither bosons or fermions.
They don't exist in nature as free particles in 3D space.
They occur as collective excitations in topologically ordered 2D systems.
What is topological order?

"Traditional" phases / transitions

Many phases / transitions can be understood in terms of symmetries and symmetry breaking.

These phases can be characterised by local order parameters



Topological phases cannot be distinguished by local order parameters. There transitions need not be described by symmetry breaking.

Types of topological phases

- Free-fermion / topological band structures
- Symmetry protected topological (SPT) e.g. SSH

} (Kane, Felser)

- Intrinsic topological order ⟵ described by TQFT
- Symmetry enriched topological order

Why do we care? (Claudia Felser)

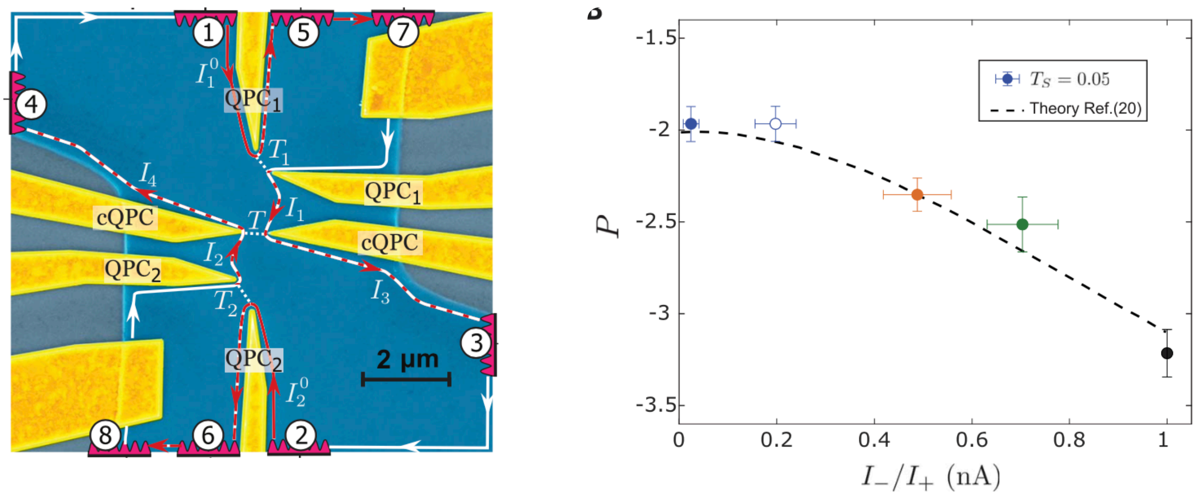
- Topological protection.
- Degenerate GS ⟷ anyon excitations.
- Quantum technologies - Quantum computers / memory.
- Questions of complexity.

  Proof TQFT ⟷ intrinsic sign-problems

Where do we get topological order / anyons ?

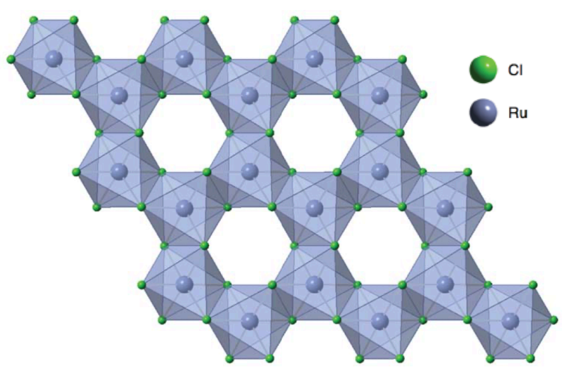
Most famously, fractional quantum hall systems are believed to be topological phases hosting (non-)abelian anyons.

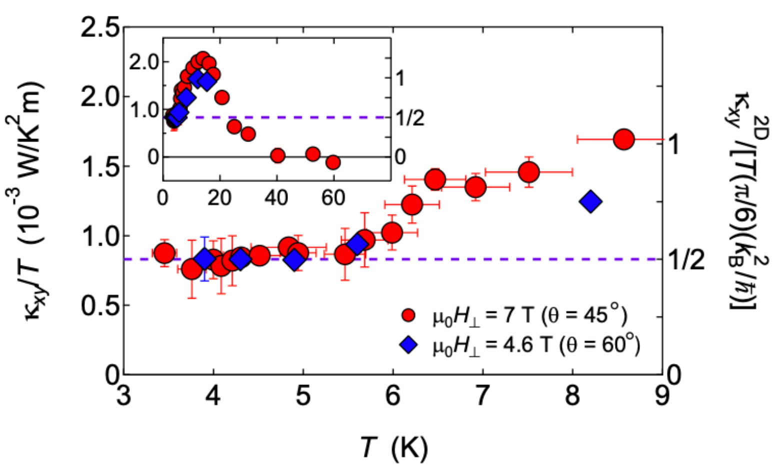
Recent "anyon collider" experiment showed presence of (abelian) anyon excitations.



Bartolomei et al Science 368, 173 (2020)

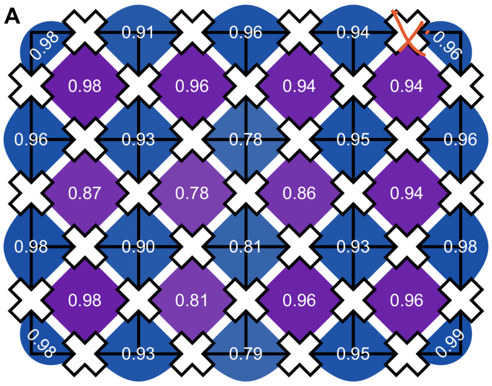
Also in magnetic systems, such as α-$RuCl_3$, which is described by the Kitaev honeycomb model with Majorana excitations.



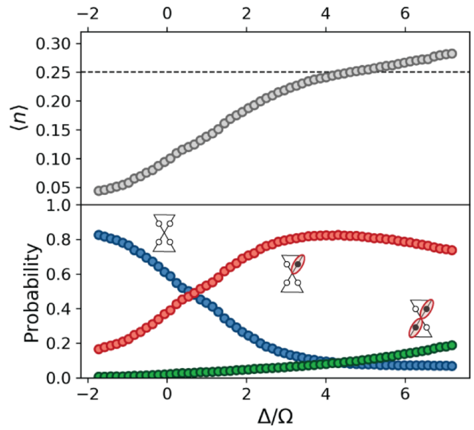


Y. Kasahara et al Nature 559, 227 (2018)

Simplest 'toy' example is Kitaev's toric code.
Recently realised experimentally by Google on a quantum computer, and by Lukin group in trapped Rydberg atoms.



Satzinger et al
Science 374, 6572 (2021)



Semeghini et al
Science 374, 6572 (2021)